ELSEVIER

Journal of Fluids and Structures 23 (2007) 143–153

JOURNAL OF FLUIDS AND STRUCTURES

www.elsevier.com/locate/jfs

An efficient reduced-order modelling approach based on fluid eigenmodes and boundary element method

H. Shahverdi[a], A.S. Nobari[a,*], M. Behbahani-Nejad[b], H. Haddadpour[c]

[a]Department of Aerospace Engineering, Amir Kabir University of Technology, Tehran 15875-4413, Iran
[b]Mechanical Engineering Department, Shahid Chamran University of Ahvaz, Ahvaz 61355-148, Iran
[c]Department of Aerospace Engineering, Sharif University of Technology, Tehran 11365-8639, Iran

Received 8 June 2005; accepted 13 June 2006
Available online 17 October 2006

Abstract

This paper presents an efficient reduced-order modelling approach based on the boundary element method. In this approach, the eigenvalue problem of the unsteady flows is defined based on the unknown wake singularities. By constructing this reduced-order model, the body quasi-static eigenmodes are removed from the eigensystem and it is possible to obtain satisfactory results without using the static correction technique when enough eigenmodes are used. In addition to the conventional method, eigenanalysis and reduced-order modelling of unsteady flows over a NACA 0012 airfoil, a wing with NACA 0012 section and a wing–body combination are performed using the proposed reduced order modelling (ROM) method. Numerical examples are presented that demonstrate the accuracy and computational efficiency of the present method.

Keywords: Eigensystem; Unsteady; Doublet; Source; Efficiency

1. Introduction

Reduced-order modelling (ROM) is a conceptually novel and computationally efficient technique that has been recently used in the analysis of unsteady flows. Unsteady flow eigenmodes are used to construct reduced-order models similar to the normal mode analysis commonly used in structural dynamics. The advantage of a modal approach is that one may construct a reduced-order model by retaining only a few of the original modes. Eigenanalysis of unsteady potential flows about flat airfoils, cascades and wings have been applied by Hall (1994). He constructed reduced-order models based on an unsteady incompressible vortex lattice method and found that to obtain satisfactory results, the static correction technique must be used. Florea and Hall (1994) created ROM in the time domain for linearized potential flow about airfoils. Also, ROM have been used for aerodynamic modelling of helicopter blades (Tang et al., 1998b) using Peters' (1994) finite state airloads model and using nonlinear aeroelastic systems (Tang et al., 1998a). Romanowski and Dowell (1996) applied ROM to subsonic unsteady flows, based on the Euler equations, around a NACA 0012 airfoil. ROM of unsteady viscous flow in a compressor cascade based on the coupled potential flow and

*Corresponding author.
E-mail addresses: Shahverdi@aut.ac.ir (H. Shahverdi), Sall358@aut.ac.ir, andishan.pars@gmail.com (A.S. Nobari), Bnmorteza@cua.ac.ir (M. Behbahani-Nejad), Haddadpour@sharif.ir (H. Haddadpour).

0889-9746/$ - see front matter
doi:10.1016/j.jfluidstructs.2006.06.008

boundary-layer approximation has been applied by Florea et al. (1998), and the status of ROM for unsteady aerodynamic systems has been reviewed by Dowell (1996) and Dowell et al. (1997).

Esfahanian and Behbahani-Nejad (2002) applied ROM to the subsonic unsteady flows about complex configurations using a boundary element method. They indicated that the zero eigenvalues of the unsteady model are equal to the number of elements that lie on the body (Behbahni-Nejad, 2002). The corresponding eigenmodes behave exactly in a quasi-static fashion, and ROM without the static correction cannot generate satisfactory results, even with a large number of eigenmodes. On the other hand, ROM based on a body and its wake eigenmodes (conventional ROM) can give satisfactory results only when the static correction technique is applied. However, when this correction is applied, the quasi-steady part of the solution must be computed for each time step, which alters the efficiency of ROM. By constructing a reduced-order model based only on the wake eigenmodes, the body quasi-static eigenmodes are removed and it is possible to obtain satisfactory results without the static correction technique. This concept has been recently applied by Behbahani-Nejad et al. (2005) for unsteady flow computations based on the vortex lattice method.

For unsteady flow computations about real and complex geometries, one needs a numerical approach other than the vortex lattice method. The boundary element method (BEM) has been known as a powerful numerical technique in engineering analysis. In CFD analysis and especially heat transfer problems, BEM plays an important and efficient role. In the beginning, this method was used in linear problems, but it developed quickly to analyze nonlinear problems too. One of its main advantages is the reduction of the problem dimensionality by one, since it will be required to discretize only the boundary of the computational domain.

The main objective of the present work is to develop an efficient ROM solver based on BEM for computation of unsteady flows around complex configurations. Coupling of the present method with modal analysis in structural dynamics will lead to a powerful tool for aeroelastic response calculations.

In this context, an alternative formulation based on the boundary element method is presented by which the eigenvalue problem is defined based only on the unknown wake singularities. Using this approach, ROM of unsteady flows about a NACA 0012 airfoil, a wing with NACA 0012 section and a wing–body combination are studied. The results show that the present reduced-order modelling approach (PROM) without static correction can produce satisfactory results when enough eigenmodes are used, depending on the properties of the system under consideration. Also, one can use more efficiently the static correction technique along with the present approach to obtain satisfactory results with a few eigenmodes. The present ROM can accurately and more efficiently analyze unsteady flows in comparison with the conventional reduced-order models (CROM).

2. Boundary element formulation

For the case of an incompressible, inviscid, irrotational flow, the Navier–Stokes equations can be reduced to the classical Laplace equation. From Green's second identity, it is shown (Katz and Plotkin, 2001) that a solution to Laplace's equation in the flow field can be expressed in integral form over the boundary surfaces, namely

$$4\pi c\phi_P = \int_{S_B}\left[\phi\frac{\partial}{\partial n}\left(\frac{1}{r}\right) - \frac{1}{r}\frac{\partial\phi}{\partial n}\right]\mathrm{d}S + \int_{S_W^U}\Delta\phi_w\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\mathrm{d}S, \tag{1}$$

where r is the distance from the point P to the boundary element dS, ϕ is perturbation velocity potential, n is the unit normal vector to the surface pointing into the flow field of interest, and S_B and S_W are the surfaces of the body and the wake, respectively.

Moreover, $c = 1/2$ if P is on a smooth part of the surface and $\Delta\phi_W$ is directly related to the velocity potential values at the trailing edge by the Kutta condition, that is,

$$\Delta\phi_W = \phi_B^U - \phi_B^L. \tag{2}$$

For two-dimensional unsteady flows, Eq. (1) reduces to (Katz and Plotkin, 2001)

$$2\pi c\phi_P = \int_{S_B}\left[\phi\frac{\partial}{\partial n}(\ln r) - \ln r\frac{\partial\phi}{\partial n}\right]\mathrm{d}S + \int_{S_W^U}\Delta\phi_W\frac{\partial}{\partial n}(\ln r)\,\mathrm{d}S. \tag{3}$$

To solve the boundary integral Eq. (1) numerically, the surfaces S_B and S_W are discretized into small quadrilateral elements, and constant-strength singularity distributions (of sources $\partial\phi/\partial n$ and doublets ϕ) are distributed on each element. Therefore, the collocation method yields the following relations for each collocation point on the body:

$$\phi_{Ph} = \sum_{k=1}^{NB} A_{hk}\phi_{Pk} + \sum_{k=1}^{NB} B_{hk}\left(\frac{\partial\phi}{\partial n}\right)_{Pk} + \sum_{k=1}^{NW} A_{hk}\,\Delta\phi_{Pk}, \tag{4}$$

where NB and NW are the number of elements on the body and the wake, respectively. The coefficients A_{hk}, B_{hk} in Eq. (4) represent the influence of the kth element singularity distribution on hth collocation point [for more details see Esfahanian and Behbahani-Nejad (2002) and Katz and Plotkin (2001)].

The second term in the right-hand side of Eq. (4) is known from the tangency condition at each time step. Moreover, $\Delta\phi$ is determined via the Kutta condition and Kelvin's theorem (Esfahanian and Behbahani-Nejad, 2002). Hence, if Eq. (4) is applied at all of the collocation points over the body and the vector μ is defined as

$$\mu = \{\phi_1, \phi_2, \ldots, \phi_{NB}, \Delta\phi_1, \Delta\phi_2, \ldots, \Delta\phi_{NW}\}, \tag{5}$$

one can obtain

$$A\mu^{n+1} + B\mu^{n} = w^{n+1}. \tag{6}$$

Given a prescribed time history of the body motion, Eq. (6) is simply marched in time from one time level (n) to the next ($n+1$) to find the time history of the strengths of the doublet elements. When the velocity potential is determined, the resulting pressure coefficient can be computed as (Katz and Plotkin, 2001)

$$C_P = 1 - \frac{V^2}{V_\infty^2} - \frac{2}{V_\infty^2}\frac{\partial\phi}{\partial t}, \tag{7}$$

where V is the local fluid velocity on the body surface.

3. Conventional reduced-order modelling

In this section, the development of ROM as given by Hall (1994) will be presented for the sake of the clarity of the paper. Consider the homogeneous part of Eq. (6), setting $\mu = x_i e^{\lambda_i t}$ and $z_i = e^{\lambda_i \Delta t}$, one obtains

$$z_i A x_i + B x_i = 0, \tag{8}$$

where λ_i and z_i are ith eigenvalues in the λ and z planes, respectively, and x_i is the corresponding eigenvector. More generally, Eq. (8) can be written as

$$AXZ + BX = 0, \tag{9}$$

where Z is a diagonal matrix containing the eigenvalues and X is a matrix with columns representing the right eigenvectors. On the other hand, the left eigenvectors satisfy

$$A^{\mathrm{T}} YZ + B^{\mathrm{T}} Y = 0, \tag{10}$$

where Y is a matrix with rows that are the left eigenvectors. If the eigenvectors are normalized suitably, they satisfy the orthogonality conditions

$$Y^{\mathrm{T}} AX = I, \tag{11}$$

$$Y^{\mathrm{T}} BX = -Z. \tag{12}$$

The dynamic behavior of the fluid flow can be represented as the sum of the individual eigenmodes, that is,

$$\mu = Xc, \tag{13}$$

where c is the vector of normal mode coordinates. Substitution of Eq. (13) into Eq. (6), premultiplying by Y^{T} and making use of the orthogonality condition gives a set of N uncoupled equations for the modal coordinates c, namely,

$$c^{n+1} - Zc^{n} = Y^{\mathrm{T}} w^{n+1}. \tag{14}$$

Now one may construct a reduced-order model by retaining only a few of the original modes. However, the preceding reduced-order model does not produce satisfactory results unless the static correction is applied (Hall, 1994). For applying the static correction technique, it is a normal procedure to decompose the unsteady solution into two parts. One part is equivalent to the response of the system if the disturbance is quasi-steady, and the other part is the dynamic part. Therefore, the unsteady solution can be defined as follows:

$$\mu^n = \mu_s^n + \mu_d^n, \tag{15}$$

$$\mu^n = \mu_s^n + X\tilde{c}^n. \tag{16}$$

The quasi-steady portion μ_s is given by

$$[A + B]\mu_s^n = w^n. \tag{17}$$

Thus, the proposed reduced-order model can be written as

$$\tilde{c}^{n+1} - Z\tilde{c}^n = Y^{\mathrm{T}} w^{n+1} - Y^{\mathrm{T}}(A\mu_s^{n+1} + B\mu_s^n). \tag{18}$$

4. Present reduced-order modelling

Pervious works (Esfahanian and Behbahani-Nejad, 2002; Behbahani-Nejad et al., 2005) have shown that the existence of zero eigenvalues in the eigensystem is the main reason for needing to apply a static correction technique. Hence, by constructing a reduced-order model based only on the wake eigenmodes, the body quasi-static eigenmodes can be removed. Here, we apply this technique for unsteady flow computations based on the boundary element method. Because of the existence of the Kutta elements (Eq. (2)) adjacent to the trailing edge, it can be shown that in the present method the number of zero eigenvalues of the unsteady model is more than the number of elements that lie on the body. Let us define μ_b as the vector of the body and the Kutta doublet strengths and μ_w as the vector of the wake doublet strengths. Using these definitions, one can write

$$\mu = \begin{Bmatrix} \mu_b \\ \mu_w \end{Bmatrix}. \tag{19}$$

Now, Eq. (6) can be written as

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \mu_b \\ \mu_w \end{Bmatrix}^{n+1} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{Bmatrix} \mu_b \\ \mu_w \end{Bmatrix}^{n} = \begin{Bmatrix} w_b \\ 0 \end{Bmatrix}^{n+1}, \tag{20}$$

or splitting into two equations

$$A_{11}\mu_b^{n+1} + A_{12}\mu_w^{n+1} + B_{11}\mu_b^n + B_{12}\mu_w^n = w_b^{n+1}, \tag{21}$$

$$A_{21}\mu_b^{n+1} + A_{22}\mu_w^{n+1} + B_{21}\mu_b^n + B_{22}\mu_w^n = 0. \tag{22}$$

It can be shown that the matrices B_{11}, B_{12} and A_{21} are zero and, therefore, Eq. (21) results in

$$\mu_b^{n+1} = A_{11}^{-1} w_b^{n+1} - A_{11}^{-1} A_{12}\mu_w^{n+1}. \tag{23}$$

Substitution of Eq. (23) into Eq. (22) gives

$$A_{\text{new}}\mu_w^{n+1} + B_{\text{new}}\mu_w^n = w_{\text{new}}^n, \tag{24}$$

where

$$A_{\text{new}} = A_{22}, \tag{25}$$

$$B_{\text{new}} = B_{22} - B_{21}A_{11}^{-1}A_{12}, \tag{26}$$

$$w_{\text{new}}^n = -B_{21}A_{11}^{-1}w_b^n. \tag{27}$$

Since Eq. (24) is only in terms of the wake doublet strengths, the corresponding eigensystem has no zero eigenvalue and one may construct accurate reduced-order models without using the static correction technique.

5. Results and discussion

5.1. Test case models

The examples presented in this section serve mainly for the validation of the proposed method and help to demonstrate the capability and efficiency of this approach. Hence, for the numerical computation, first a NACA 0012 airfoil is considered as a two-dimensional test case. The airfoil is modelled using 72 boundary elements with cosine distribution. The wake length is taken to be 10 times the chord length and it is discretized using 100 elements. Also, a three-dimensional wing with the NACA 0012 airfoil section and the aspect ratio 4.0, is used as a three-dimensional test case (Fig. 1). The surface of the wing is modelled using 20 elements in both the chordwise and spanwise directions. The wake length is taken to be 10 times the chord length and discretized using 40 elements in streamwise direction. As the wing is symmetric, only half of the wing is modelled in the computational domain.

To show the capability of the PROM using BEM in unsteady flow analysis over complex configurations, a wing–body combination is considered as shown in Fig. 2. Fig. 3 shows the computational mesh over the wing–body and its wake. Because of symmetry, half of the wing–body and its wake are discretized. The surface of this model is discretized using 1450 elements. The wake length is taken to be 10 times the maximum chord length and is discretized using 20 and 40 elements in the spanwise and streamwise directions, respectively.

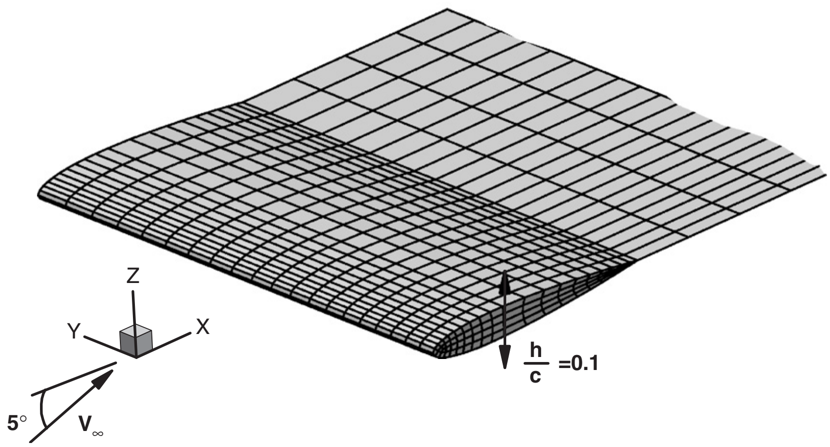

Fig. 1. Geometry of the three-dimensional wing and its wake in heaving oscillation (half of the wing is shown).

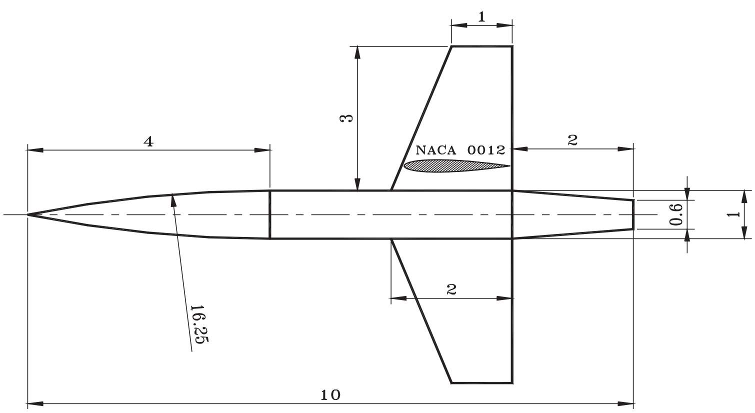

Fig. 2. Dimensions of the wing–body combination.

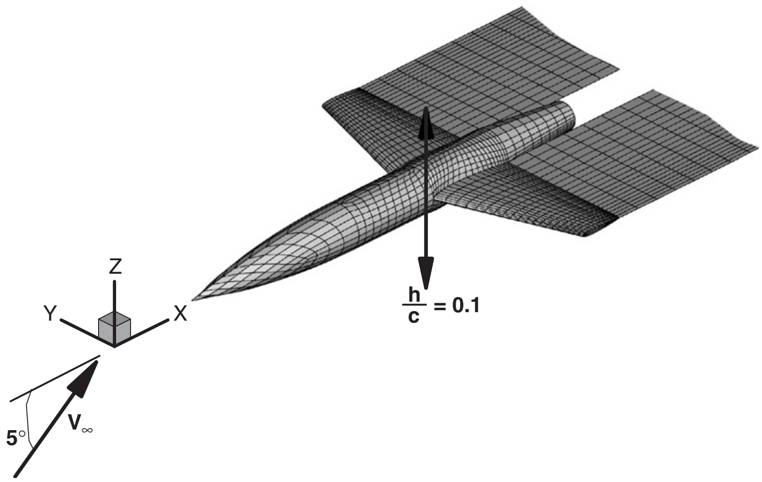

Fig. 3. Geometry of the wing–body combination and its wake in heaving oscillation.

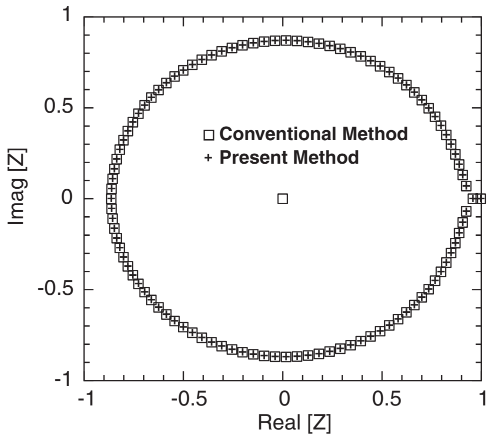

Fig. 4. Eigenvalues of boundary element model of unsteady flow about the NACA 0012 airfoil.

5.2. Eigenanalysis

The results of the conventional and present eigenanalysis are discussed in this section. The eigenvalues and eigenvectors are computed using well-known IMSL routines (IMSL, 1980). Eigenvalues of the present method are shown in Fig. 4 in comparison with those of the conventional method for the two-dimensional airfoil. The results show that the eigenvalues of the present method are the same as the nonzero eigenvalues of the conventional method. In the present eigenanalysis, the eigensystem is comprised of wake elements, except for the Kutta elements. Therefore, there are no zero eigenvalues related to the body elements. On the other hand, in the conventional method, the eigensystem is constructed using the body elements, as well as the wake elements. Therefore, there are 73 zero eigenvalues related to the body and the Kutta elements, and 99 nonzero eigenvalues related to the wake elements.

Eigenvalues of the unsteady flow about the three-dimensional wing and wing–body combination are plotted in Figs. 5 and 6, respectively. As shown in these figures, the nonzero eigenvalues of the conventional method are the same as the eigenvalues of the present method. However, as is shown in Figs. 5 and 6, relatively large differences exist between high-frequency eigenvalues of the conventional and present eigensystems. Our numerical experience shows that the IMSL eigensystem routines are sensitive to computational errors, e.g., roundoff errors, especially when the dimension of the corresponding eigensystem is large. This sensitivity results in relatively large errors in the computation of the high-frequency eigenvalues. Fortunately, these errors do not affect the CROM and PROM features, as both methods use only low-frequency eigenmodes.

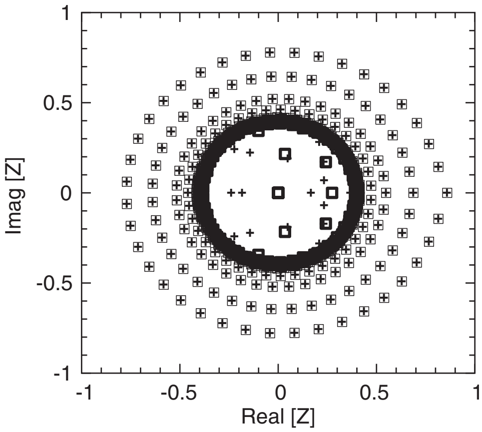

Fig. 5. Eigenvalues of boundary element model of unsteady flow about the three-dimensional wing: □, conventional method; +, present method.

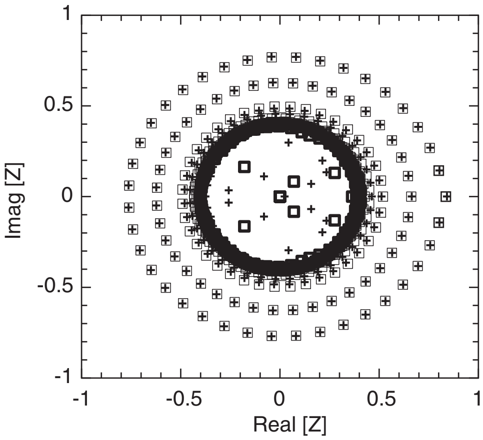

Fig. 6. Eigenvalues of boundary element model of unsteady flow about the wing-body combination: □, conventional method; +, present method.

5.3. Reduced-order models

Next, we use the eigenmode information computed in the pervious section to construct the present reduced-order aerodynamic model. Fig. 7 shows typical reduced-order model results for the pitch oscillation of the airfoil. The pitching axis is at the quarter chord of the airfoil, the angle of attack, $\alpha = 3^\circ + 10^\circ \sin kT$ and the reduced frequency, $k = 0.10$. Although the angle of attack varies in a wide range ($-7^\circ \leqslant \alpha \leqslant 13^\circ$) and the wake roll-up is not considered, comparison of the present method with Katz and Maskew's (1988) unsteady panel method is satisfactory. Also, the figure illustrates PROM results in comparison with the direct method. Although PROM with 40 modes without static correction is not capable of producing a suitable result especially in the pitching moment calculation, using the static correction technique with just four modes, it results in a very good agreement with the direct method. However, if sufficient eigenmodes are used, the static correction requirement will be removed. Figs. 8 and 9 present the results of the airfoil oscillating at $\alpha = \pm 1^\circ$ and Mach 0.50, with reduced frequencies $k = 0.10$ and 0.40, respectively. The Prandtl–Glauert compressibility correction is used to consider compressibility effects. The present BEM and PROM results are in excellent agreement with the unsteady Euler solution used in Romanowski (1995). Also, the results show that PROM without static correction can produce satisfactory results when enough eigenmodes are used.

As is shown in Fig. 1, the three-dimensional wing oscillates with an amplitude of $h/c = 0.10$ about a 5° angle of attack. Computed results for the lift variation during a heaving oscillation cycle with reduced frequencies $k = 0.10$ and

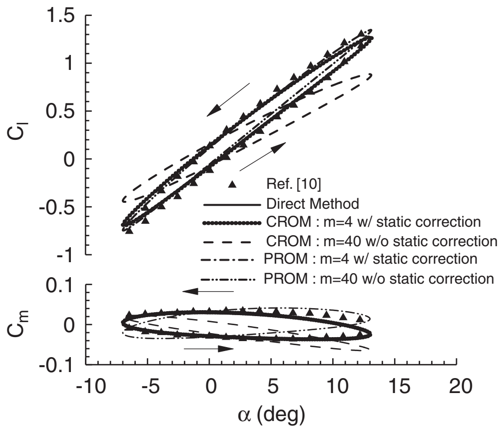

Fig. 7. Lift and pitching moment loops for the pitch oscillation of a NACA 0012 airfoil.

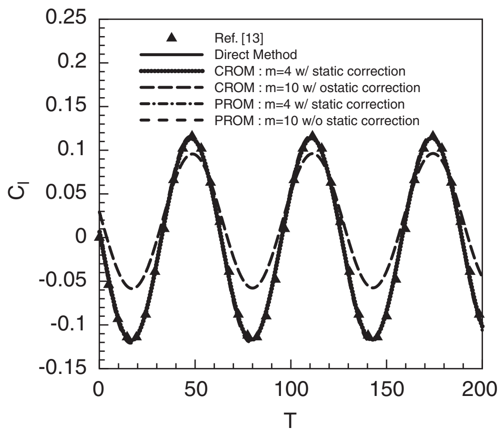

Fig. 8. Lift response versus time for a NACA 0012 airfoil at Mach 0.5 oscillating at $\alpha = \pm 1^\circ$ about zero angle of attack with $k = 0.10$.

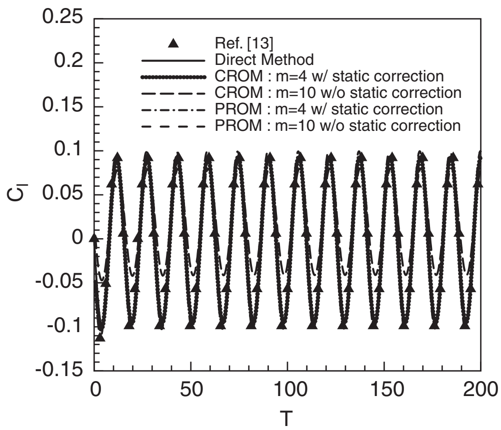

Fig. 9. Lift response versus time for a NACA 0012 airfoil at Mach 0.5 oscillating at $\alpha = \pm 1^\circ$ about zero angle of attack with $k = 0.40$.

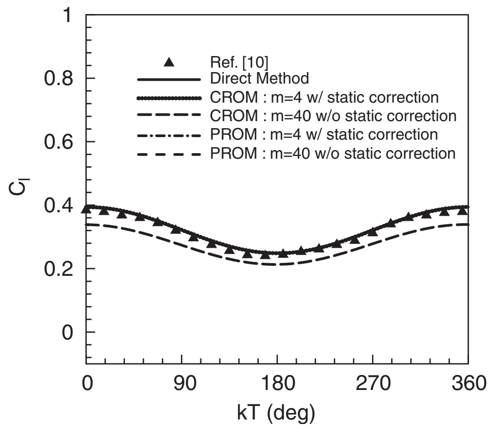

Fig. 10. Lift variation during heaving oscillation with $k = 0.01$, of a three-dimensional wing of NACA 0012 section.

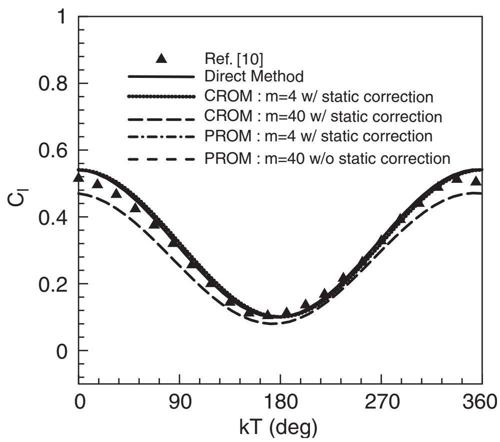

Fig. 11. Lift variation during heaving oscillation with $k = 0.30$, of a three-dimensional wing of NACA 0012 section.

0.30 are shown in Figs. 10 and 11, respectively. The results the direct method are in excellent agreement with those presented in Katz and Maskew (1988). However, the figures show CROM and PROM results with four eigenmodes with the static correction and results with 40 eigenmodes without static correction. As expected, PROM without static correction shows good agreement with the direct method when enough eigenmodes are used. In addition, PROM along with static correction gives satisfactory results with just a few eigenmodes.

Also, computed results for the lift variation of the wing–body combination during a heaving oscillation cycle with an amplitude of $h/c = 0.10$ about a 5° angle of attack and reduced frequency $k = 0.30$ are presented in Fig. 12. The results of CROM and PROM with four eigenmodes with static correction and with 40 eigenmodes without static correction are compared with those of the direct method. The results show a good agreement between PROM and CROM with static correction. As expected, PROM with enough eigenmodes and without static correction technique shows an excellent agreement with the direct method. Moreover, PROM along with static correction gives satisfactory results with only a few eigenmodes.

5.4. Efficiency analysis

Finally, the efficiency of PROM is discussed. To clarify the efficiency analysis, CPU times for PROM and CROM with static correction technique, along with four eigenmodes, are compared. In addition, CPU times for eigenvalue computations and ROM are presented separately. Table 1 indicates CPU times in seconds for the two-dimensional airfoil, three-dimensional wing and wing–body combination. The results are based on numerical computations using a

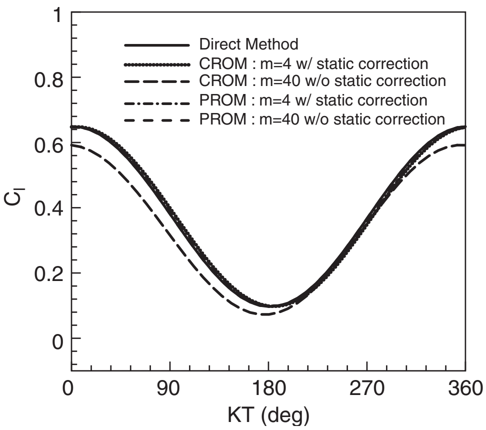

Fig. 12. Lift variation during heaving oscillation with $k = 0.30$, of the wing–body combination.

Table 1
CPU time (s) comparison between conventional and present method

Test case	Number of elements		Eigenanalysis		ROM			
	Body	Wake	Conventional	Present	Conventional	Conventional W LU	Present	Present W LU
Airfoil	72	100	0.3	0.1	8.6	—	3.8	—
Wing	872	800	380.5	102.7	737.5	—	173.3	—
Wing–body	1450	800	717.8	125.2	1430.8	870.3	221.1	116.3

P4-3200 MHz with 2-GB RAM. The results presented in Table 1 reveal that, regardless of the application of the LU decomposition, the present method can analyze either eigensystem or ROM more efficiently than CROM with static correction. The efficiency of the present method is due to the fact that the resulting eigensystem has a smaller dimension than the conventional method, since it is represented based only on the wake elements. Therefore, the present method will be more efficient when the ratio of the number of body elements to the number of wake elements is increased. However, the application of the present method without the static correction technique is more efficient than CROM because there is no need to compute the quasi-steady solution in each time step when enough eigenmodes are used. Hence, the present method will be more efficient as time increases in time domain analysis.

6. Conclusions

This study demonstrates that PROM of unsteady flows in general two- and three-dimensional cases is more efficient than the CROM. In the present method, the numerical eigensystem is only constructed using the wake singularities, which results in the smaller dimension of the corresponding eigensystem. It was shown that the present reduced-order model without the static correction technique can produce satisfactory results when enough eigenmodes are used. In addition, one can use more efficiently the static correction technique along with the present approach to obtain satisfactory results with a few eigenmodes. Finally, based on the present results, it can be concluded that the present method is computationally more efficient than the conventional method. This efficiency is more evident for unsteady flow computations around a complex configuration.

Acknowledgments

Amir Kabir University of Technology, Shahid Chamran University of Ahvaz and Sharif University of Technology are acknowledged for providing technical, administrative and financial assistance.

References

Behbahni-Nejad, M., 2002. Reduced order modeling of 3D unsteady flows using fluid eigenmodes and boundary element method. Ph.D. Dissertation, University of Tehran, Tehran, Iran.

Behbahani-Nejad, M., Haddadpour, H., Esfahanian, V., 2005. Reduced-order modeling of unsteady flows without static correction requirement. Journal of Aircraft 42 (4), 882–886.

Dowell, E.H., 1996. Eigenmode analysis in unsteady aerodynamics: reduced order models. AIAA Journal 34 (8), 1578–1588.

Dowell, E.H., Hall, K.C., Romanowski, M.C., 1997. Eigenmode analysis in unsteady aerodynamics: reduced order models. Applied Mechanics Reviews 50 (6), 371–385.

Esfahanian, V., Behbahani-Nejad, M., 2002. Reduced order modeling of unsteady flows about complex configurations using the boundary element method. ASME Journal of Fluids Engineering 124 (4), 988–993.

Florea, R., Hall, K.C., 1994. Reduced order modeling of unsteady flows about airfoils. In: Friedmann, P.P., Chang, J.C.I. (Eds.), Symposium on Aeroelasticity and Fluid/Structure Interaction. Winter Annual Meeting, Chicago, ASME Press, New York.

Florea, R., Hall, K.C., Cizmas, P.G., 1998. Reduced-order modeling of unsteady viscous flow in a compressor cascade. AIAA Journal 36, 1039–1048.

Hall, K.C., 1994. Eigenanalysis of unsteady flows about airfoils, cascades, and wings. AIAA Journal 32, 2426–2432.

IMSL library reference manual, 1980, IMSL LIB-0008.

Katz, J., Maskew, B., 1988. Unsteady low-speed aerodynamic model for complete aircraft configurations. Journal of Aircraft 25 (4), 302–310.

Katz, J., Plotkin, A., 2001. Low-Speed Aerodynamics From Wing Theory to panel Methods, second ed. Cambridge University Press, New York.

Peters, D.A., 1994. Finite state airloads for deformable airfoils on fixed and rotating wings. In: Friedmann, P.P., Chang, J.C.I. (Eds.), Symposium on Aeroelasticity and Fluid/Structure Interaction. Winter Annual Meeting, Chicago, ASME Press, New York.

Romanowski, M.C., 1995. Using fluid eigenmodes to develop Euler based reduced order unsteady aerodynamic and aeroelastic modes. Ph.D. Thesis, Duke University, North Carolina.

Romanowski, M.C., Dowell, E.H., 1996. Reduced order Euler equations for unsteady aerodynamic flows: numerical techniques. AIAA paper 96-0528.

Tang, D., Conner, M.D., Dowell, E.H., 1998a. Reduced order aerodynamic model and its application to a nonlinear aeroelastic system. Journal of Aircraft 35 (2), 332–338.

Tang, D., Dowell, E.H., Peters, D.A., 1998b. Reduced order aerodynamics models based upon inflow eigenmodes. Journal of the American Helicopter Society 43 (4), 342–351.